

Chapter 7: Cubes and Cube Roots

Question 1

Find the cubes of the following numbers: (i) 7, (ii) 12, (iii) 21, (iv) 100, (v) 302

Solution:

(i) $(7)^3 = 7 \times 7 \times 7 = 343$

(ii) $(12)^3 = 12 \times 12 \times 12 = 1728$

(iii) $(21)^3 = 21 \times 21 \times 21 = 9621$

(iv) $(100)^3 = 100 \times 100 \times 100 = 1000000$

(v) $(302)^3 = 302 \times 302 \times 302 = 27543608$

Question 2

Write cubes of all natural numbers between 1 and 20 and verify the following statements:

- (a) Cubes of all odd natural numbers are odd.
- (b) Cubes of all even natural numbers are even.

Solution:

$(2)^3 = 8, (3)^3 = 27, (4)^3 = 64, (5)^3 = 125, (6)^3 = 216, \dots, (19)^3 = 6859.$

- (a) Yes, cubes of all odd natural numbers are odd.
- (b) Yes, cubes of all even natural numbers are even.

Question 3

Write cubes of 5 natural numbers which are multiples of 3 and

verify the following:

'The cube of natural number, which is a multiple of 3 is a multiple of 27'.

Solution:

$$(3)^3 = 3 \times 3 \times 3 = 27$$

$$(6)^3 = 6 \times 6 \times 6 = 216$$

$$(9)^3 = 9 \times 9 \times 9 = 729$$

$$(12)^3 = 12 \times 12 \times 12 = 1728$$

$$(15)^3 = 15 \times 15 \times 15 = 3375$$

Verification:

$$(3)^3 = 27 = 27 \times 1$$

$$(6)^3 = 216 = 27 \times 8$$

$$(9)^3 = 729 = 27 \times 27$$

$$(12)^3 = 1728 = 27 \times 64$$

$$(15)^3 = 3375 = 27 \times 125$$

∴ The cube of natural number, which is a multiple of 3 is a multiple of 27'.

Question 4

Write cubes of 5 natural numbers which are of the form $3n+1$ (e.g 4, 7, 10, ...) and verify the following: 'The cube of a natural number of the form $3n + 1$ is a natural number of the same form'.

Solution:

The 5 natural numbers which are of the form $3n + 1$ (e.g 4, 7, 10, ...) are as follows:

$$3 \times 1 + 1 = 3 + 1 = 4$$

$$3 \times 2 + 1 = 6 + 1 = 7$$

$$3 \times 3 + 1 = 9 + 1 = 10$$

$$3 \times 4 + 1 = 12 + 1 = 13$$

$$3 \times 5 + 1 = 15 + 1 = 16$$

The cubes of 5 natural numbers which are of the form $3n + 1$ (e.g. 4, 7, 10, ...) are as follows:

$$(4)^3 = 4 \times 4 \times 4 = 64$$

$$(7)^3 = 7 \times 7 \times 7 = 343$$

$$(10)^3 = 10 \times 10 \times 10 = 1000$$

$$(13)^3 = 13 \times 13 \times 13 = 2197$$

$$(16)^3 = 16 \times 16 \times 16 = 4096$$

Verification:

$$64 = 3 \times 21 + 1$$

$$343 = 3 \times 114 + 1$$

$$1000 = 3 \times 333 + 1$$

$$2197 = 3 \times 732 + 1$$

$$4096 = 3 \times 1365 + 1$$

\therefore 'The cube of a natural number of the form $3n + 1$ is a natural number of the same form'.

Question 5

Write cubes of 5 natural numbers which are of the form $3n + 2$ (e.g. 5, 8, 11, ...) and verify the following: 'The cube of a natural number of the form $3n + 2$ is a natural number of the same form'.

Solution:

The 5 natural numbers which are of the form $3n + 2$ (e.g. 5, 8, 11, ...) are as follows:

$$3 \times 1 + 2 = 3 + 2 = 5$$

$$3 \times 2 + 2 = 6 + 2 = 8$$

$$3 \times 3 + 2 = 9 + 2 = 11$$

$$3 \times 4 + 2 = 12 + 2 = 14$$

$$3 \times 5 + 2 = 15 + 2 = 17$$

The cubes of 5 natural numbers which are of the form $3n + 2$ (e.g. 5, 8, 11, ...) are as follows:

$$(5)^3 = 5 \times 5 \times 5 = 125$$

$$(8)^3 = 8 \times 8 \times 8 = 512$$

$$(11)^3 = 11 \times 11 \times 11 = 1331$$

$$(14)^3 = 14 \times 14 \times 14 = 2744$$

$$(17)^3 = 17 \times 17 \times 17 = 4913$$

Verification:

$$125 = 3 \times 41 + 2$$

$$512 = 3 \times 170 + 2$$

$$1331 = 3 \times 443 + 2$$

$$2744 = 3 \times 914 + 2$$

$$4913 = 3 \times 1637 + 2$$

∴ 'The cube of a natural number of the form $3n + 2$ is a natural number of the same form'.

Which of the following numbers are perfect cubes? 1728, 106480

Solution:

$$\begin{array}{r|l} 2 & 1728 \\ \hline 2 & 864 \\ \hline 2 & 432 \\ \hline 2 & 216 \\ \hline 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$= (2^3 \times 2^3 \times 3^3) = (2 \times 2 \times 3)^3$$

$$\text{Therefore cube root of } 1728 = \sqrt[3]{1728} = 12$$

Hence 1728 is a perfect cube.

$$\begin{array}{r}
 2 \overline{)106480} \\
 \underline{2} \quad 53240 \\
 2 \overline{)26620} \\
 \underline{2} \quad 13310 \\
 5 \overline{)6655} \\
 \underline{11} \quad 1331 \\
 11 \overline{)1331} \\
 \underline{11} \quad 121 \\
 \underline{11} \quad 11
 \end{array}$$

$$106480 = 2 \times 2 \times 2 \times 5 \times 11 \times 11 \times 11$$

$$= (2^3 \times 2 \times 5 \times 11^3) = (2 \times 11)^3 \times 2 \times 5$$

In the above factorisation 2×5 remains after grouping in triplets. Therefore 106480 is not a perfect cube.

Question 7

What is the smallest number by which 392 must be multiplied so that the product is a perfect cube ?

Solution:

$$392 = 2 \times 2 \times 2 \times 7 \times 7$$

7 occurs as a prime factor only twice.

Hence, 7 is the smallest number by which 392 must be multiplied so that the product is a perfect cube.

$$\begin{array}{r}
 2 \overline{)392} \\
 \underline{2} \quad 196 \\
 2 \overline{)98} \\
 \underline{7} \quad 49 \\
 \underline{7} \quad 7
 \end{array}$$

Question 8

What is the smallest number by which 8640 must be divided so that

the quotient is a perfect cube ?

Solution:

$$8640 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$$

5 occurs as a prime number only once.

Hence, 5 is the smallest number by which 8640 must be divided so that the quotient is a perfect cube.

$$\begin{array}{r} 2 \overline{) 8640} \\ 2 \overline{) 4320} \\ 2 \overline{) 2160} \\ 2 \overline{) 1080} \\ 2 \overline{) 540} \\ 2 \overline{) 270} \\ 3 \overline{) 135} \\ 3 \overline{) 45} \\ 3 \overline{) 15} \\ 5 \end{array}$$

Question 9

If one side of a cube is 13 metres, find its volume.

Solution:

$$\text{The volume of a cube} = (\text{side})^3 = (13)^3 = 2197\text{m}^3.$$

Question 10

Find the cube root of:

- (i) 343 (ii) 1000 (iii) 2744 (iv) 74088

Solution:

(i) $343 = 7 \times 7 \times 7$

$$\begin{array}{r|l} 7 & 343 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

$\therefore \sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$

(ii) $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

$$\begin{array}{r|l} 2 & 1000 \\ \hline 2 & 500 \\ \hline 2 & 250 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$\therefore \sqrt[3]{1000} = \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5} = 2 \times 5 = 10$

(iii) $2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$

$$\begin{array}{r|l} 2 & 2744 \\ \hline 2 & 1372 \\ \hline 2 & 686 \\ \hline 7 & 343 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

$\therefore \sqrt[3]{2744} = \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7} = 2 \times 7 = 14$

(iv) $74088 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
	7

$$\therefore \sqrt[3]{74088} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7} = 2 \times 3 \times 7 = 42$$

Question 11

Find the cube root of 125.

Solution:

$$\begin{array}{r}
 125 \\
 - 1 \\
 \hline
 124 \\
 - 7 \\
 \hline
 117 \\
 - 19 \\
 \hline
 98 \\
 - 37 \\
 \hline
 61 \\
 - 61 \\
 \hline
 0
 \end{array}$$

Since we had to subtract five times, therefore, $\sqrt[3]{125} = 5$

Question 12

Multiply 137592 by the smallest number so that the product is a perfect cube. Also, find the cube root of the product.

Solution:

$$137592 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 13$$

The number 7 and 13 should be multiplied once and twice respectively so that the product is a perfect cube.

∴ The smallest number by which 137592 must be multiplied = $7 \cdot 13 \cdot 13 = 1183$

$$\begin{aligned} \text{The required product} &= 137592 \times 1183 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 13 \times 7 \times 13 \times 13 \\ &= (2^3 \times 3^3 \times 7^3 \times 13^3) \\ &= (2 \times 3 \times 7 \times 13)^3 \\ \sqrt[3]{137592 \times 1183} &= 2 \times 3 \times 7 \times 13 \\ &= 546 \end{aligned}$$

Question 13

Divide the number 26244 by the smallest number so that the quotient is a perfect cube. Also, find the cube root of the quotient.

Solution:

$$\begin{array}{r} 2 \overline{) 26244} \\ 2 \overline{) 13122} \\ 3 \overline{) 6561} \\ 3 \overline{) 2187} \\ 3 \overline{) 729} \\ 3 \overline{) 243} \\ 3 \overline{) 81} \\ 3 \overline{) 27} \\ 3 \overline{) 9} \\ 3 \overline{) 3} \end{array}$$

$$26244 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

$2 \times 2 \times 3 \times 3 = 36$ is the smallest number by which 26244 must be divided so that the quotient is a perfect cube.

$$\begin{array}{r}
 729 \\
 36 \overline{)26244} \\
 \underline{252} \\
 0104 \\
 \underline{0072} \\
 00324 \\
 \underline{00324} \\
 00000
 \end{array}$$

The required quotient is = 729

The cube root of the quotient = $\sqrt[3]{729} = \sqrt[3]{9 \times 9 \times 9} = 9$

Question 14

The volume of a cube is 512 cubic metres. Find the length of the side of the cube.

Solution:

We know that, the volume of a cube = (side)³

The length of the side of a cube = $\sqrt[3]{512} = \sqrt[3]{2 \times 2 \times 2} = 2 \times 2 \times 2 = 8 \text{ m}$

Question 15

Which of the following numbers are cubes of negative integers?

- (a) -64 (b) -2197 (c) -1056 (d) -3888

Solution:

(a) $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$$\sqrt[3]{64} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2} = 2 \times 2 = 4$$

$$\sqrt[3]{-64} = -\sqrt[3]{64} = -4$$

∴ -64 is a cube of -4 a negative integer.

(b) $2197 = 13 \times 13 \times 13$

$$\sqrt[3]{2197} = \sqrt[3]{13 \times 13 \times 13} = 13$$

$$\sqrt[3]{-2197} = -\sqrt[3]{2197} = -13$$

∴ -2197 is a cube of -13 a negative integer.

(c) $1056 = 2 \times 2 \times 2 \times 2 \times 3 \times 11$

In the above factorisation $2 \times 3 \times 3 \times 11 \times 11$ remains after grouping in triplets. Therefore, 1056 is not a perfect cube.

Hence -1056 is not a cube of negative integer.

(d) $3888 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3$

In the above factorisation $2 \times 3 \times 3$ remains after grouping in triplets. Therefore, 3888 is not a perfect cube.

Hence -3888 is not a cube of negative integer.

Question 16

Find the cube roots of:

(a) -125 (b) -5832 (c) -17576

Solution:

(a) $125 = 5 \times 5 \times 5$

$$\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$$

$$\sqrt[3]{-125} = -\sqrt[3]{125} = -5$$

(b) $5832 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3$

$$\sqrt[3]{5832} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3} = 2 \times 3 \times 3 = 18$$

$$\sqrt[3]{-5832} = -\sqrt[3]{5832} = -18$$

(c) $17576 = 2 \times 2 \times 2 \times 13 \times 13 \times 13$

$$\sqrt[3]{17576} = \sqrt[3]{2 \times 2 \times 2 \times 13 \times 13 \times 13} = 2 \times 13 = 26$$

$$\sqrt[3]{-17576} = -\sqrt[3]{17576} = -26$$

Find the cube root of each of the following numbers:

1. 8×64
2. $(-216) \times 1728$
3. $27 \times (-2744)$
4. $(-125) \times (-3375)$
5. -456533
6. -5832000

Solution:

$$(1) \quad 8 \times 64 = 2 \times 2$$

$$\sqrt[3]{8 \times 64} = 2 \times 2 \times 2 = 8$$

$$(2) \quad 216 \times 1728 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$\sqrt[3]{(-216) \times 1728} = -(2 \times 3 \times 2 \times 2 \times 3) = -72$$

$$(3) \quad 27 \times 2744 = 3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7$$

$$\sqrt[3]{27 \times (-2744)} = -(3 \times 2 \times 7) = -42$$

$$(4) \quad 125 \times 3375 = 5 \times 5 \times 5 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

$$\sqrt[3]{(-125) \times (-3375)} = 5 \times 3 \times 5 = 75$$

$$(5) \quad 456533 = 7 \times 7 \times 7 \times 11 \times 11 \times 11$$

$$\sqrt[3]{-456533} = -(7 \times 11) = -77$$

$$(6) \quad 5832000 = 5832 \times 1000 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$$

$$\sqrt[3]{(-5832) \times 1000} = -(2 \times 3 \times 3 \times 2 \times 5) = -180$$

Question 18

Find the cubes of the following by multiplication.

- (i) -4
- (ii) 23
- (iii) 3030

Solution:

(i) $(-4)^3 = (-4) \times (-4) \times (-4) = -64$

(ii) $(23)^3 = 23 \times 23 \times 23 = 12167$

(iii) $(3030)^3 = 3030 \times 3030 \times 3030 = 27818127000$.

Question 19

Find the cube of the following rational numbers:

(i) 1.4

Solution:

(i) $(1.4)^3 = 1.4 \times 1.4 \times 1.4 = 2.744$.

Question 20

By what number would you multiply 231525 to make it a perfect cube?

Solution:

The prime factorisation of 231525 is $5 \times 5 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$.

The number that must be multiplied in order that the above product is a perfect cube is 5.

Therefore, Cube root of 231525×5 is $5 \times 3 \times 7 = 105$.